

Unit 5 Review – Analytical Applications of Differentiation

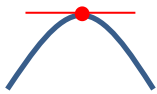
This review summarizes everything from Unit 5 along with examples but contains no problems to work through.

DEFINITIONS

Extrema: The maximum and minimum points. Extrema can be absolute or relative.

Critical Points: Where the first derivative is zero or DNE. These are possible maximum, minimum, or points of inflection!

$$f'(x) = 0$$



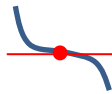
Horizontal
Tangent
Maximum

$$f'(x) = 0$$



Horizontal
Tangent
Minimum

$$f'(x) = 0$$



Horizontal
Tangent
Point of Inflection

$$f'(x) = DNE$$



Vertical Tangent
Point of Inflection

$$f'(x) = DNE$$



Cusp (No
Tangent)
Maximum

Concavity: Where the function is “cupping” up or down

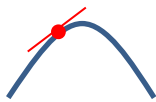


Points of Inflection: Where the second derivative is zero or DNE and changes sign!

FIRST DERIVATIVE

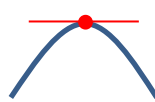
The first derivative is the instantaneous rate of change, or the slope of the tangent line, and can determine if the function is increasing or decreasing at a given point.

$$f'(x) > 0$$



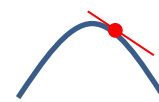
Function is increasing

$$f'(x) = 0$$



Function is not increasing
or decreasing

$$f'(x) < 0$$



Function is decreasing

SECOND DERIVATIVE

The second derivative determines concavity.

$$f''(x) > 0$$



Concave Up

$$f''(x) = 0$$

Neither concave up or
concave down

$$f''(x) < 0$$



Concave Down

FINDING EXTREMA

The First Derivative Test

STEPS	EXAMPLE												
1. Find the critical points.	$f(x) = x^2 + 2x + 1$ $f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$												
2. Determine whether the function is increasing or decreasing on each side of every critical point. A chart or number line helps!	<table border="1"> <thead> <tr> <th>Interval</th> <th>$(-\infty, -1)$</th> <th>-1</th> <th>$(-1, \infty)$</th> </tr> </thead> <tbody> <tr> <td>Test Value</td> <td>-2</td> <td>-1</td> <td>2</td> </tr> <tr> <td>$f'(x)$</td> <td>$f'(-2) = -$ Negative</td> <td>$f'(-1) = 0$</td> <td>$f'(2) = 6$ Positive</td> </tr> </tbody> </table> <p>Function decreases to the left and increases to the right of $x = -1$ so it must be relative minimum point</p>	Interval	$(-\infty, -1)$	-1	$(-1, \infty)$	Test Value	-2	-1	2	$f'(x)$	$f'(-2) = -$ Negative	$f'(-1) = 0$	$f'(2) = 6$ Positive
Interval	$(-\infty, -1)$	-1	$(-1, \infty)$										
Test Value	-2	-1	2										
$f'(x)$	$f'(-2) = -$ Negative	$f'(-1) = 0$	$f'(2) = 6$ Positive										

The Second Derivative Test

STEPS	EXAMPLE
1. Find the critical points.	$f(x) = x^2 + 2x + 1$ $f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Determine whether the function is concave up or concave down at every critical point using the second derivative.	$f''(-1) = 2$ Second derivative is positive at $x = -1$ Concave up $x = -1$ is a relative minimum point

Finding Absolute Extrema on an Interval (Candidates Test)

STEPS	EXAMPLE
1. Find the critical points. The critical points are candidates as well as the endpoints of the interval.	$f(x) = x^2 + 2x + 1$ on the interval $[-3, 0]$ $f'(x) = 2x + 2$ $0 = 2x + 2$ $x = -1$
2. Check all candidates using the $f(x)$.	$f(-3) = 4$ absolute maximum $f(-1) = 0$ absolute minimum $f(0) = 1$

LINEAR MOTION

The chart matches up function vocabulary with linear motion vocabulary.

FUNCTION	LINEAR MOTION
Value of a function at x	Position at time t
First Derivative	Velocity
Second Derivative	Acceleration
$f'(x) > 0$ Increasing Function	Moving right or up
$f'(x) < 0$ Decreasing Function	Moving left or down
$f'(x) = 0$	Not moving
Absolute Max	Farthest right or up
Absolute Min	Farthest left or down
$f'(x)$ changes signs	Object changes direction
$f'(x)$ and $f''(x)$ have same sign	Speeding Up
$f'(x)$ and $f''(x)$ have different signs	Slowing Down

Example:

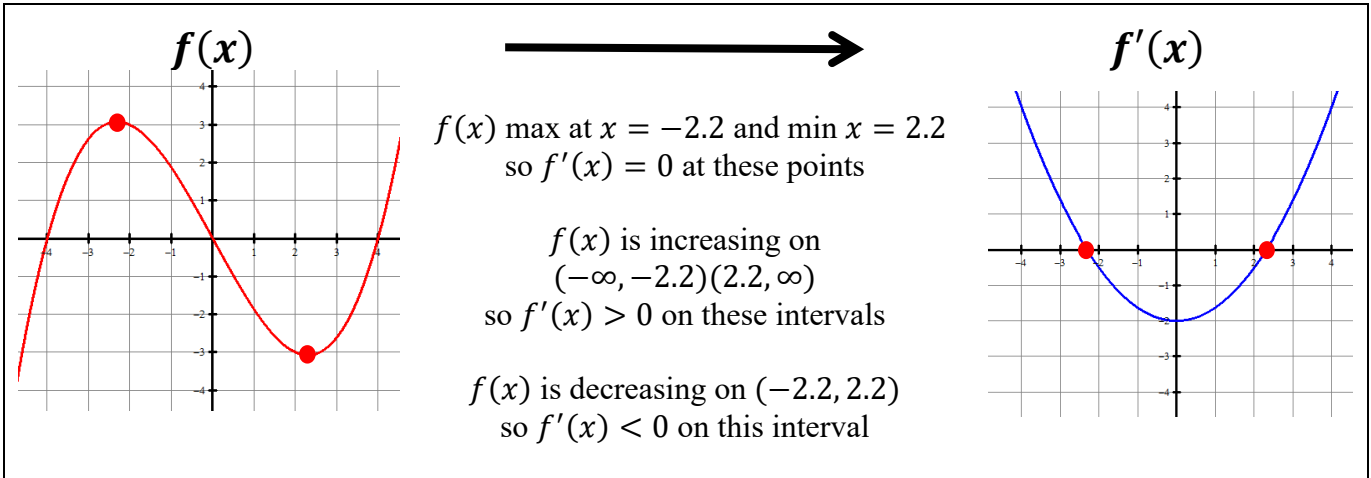
A particle moves along the x -axis with the position function $x(t) = t^4 - 4t^3 + 2$ where $t > 0$.

Interval	(0, 2)	2	(2, 3)	3	(3, ∞)
$x'(t)$ velocity	$x'(t) < 0$ decreasing left	$x'(t) < 0$ decreasing left	$x'(t) < 0$ decreasing left	$x'(t) = 0$ Not moving	$x'(t) > 0$ Increasing right
$x''(x)$ acceleration	$x''(t) < 0$ Concave down	$x''(t) = 0$	$x''(t) > 0$ Concave up	$x''(t) > 0$ Concave up	$x''(t) > 0$ Concave up
Conclusion	Speeding Up	Moving Left	Slowing Down	Not Moving	Speeding Up

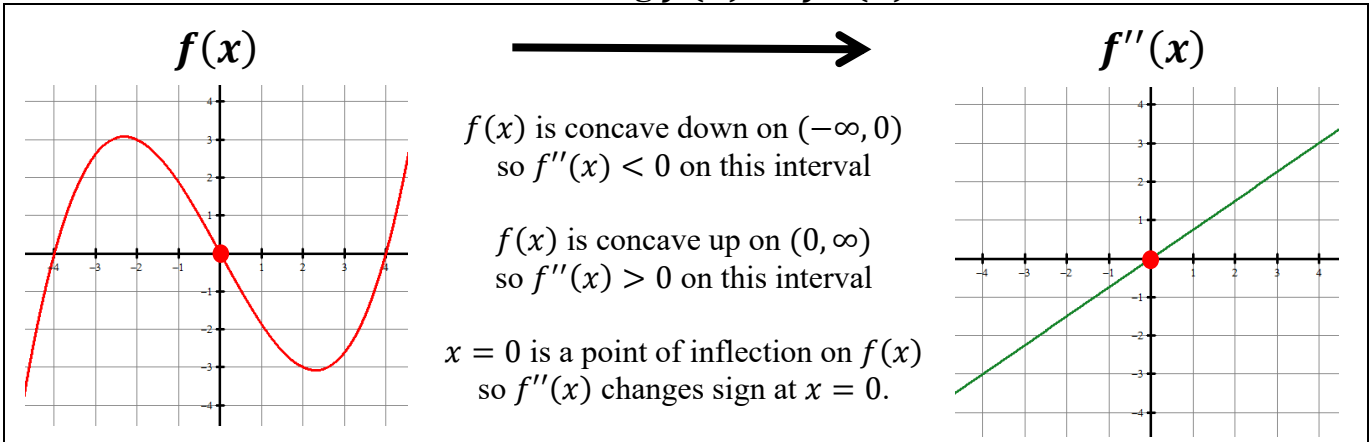
FUNCTION	LINEAR MOTION
$t = 3$ is minimum	$t = 3$ has no velocity Changing direction
Decreasing (0,3)	Moving left (0,3)
Increasing (3, ∞)	Moving right (3, ∞)

GRAPHICAL ANALYSIS

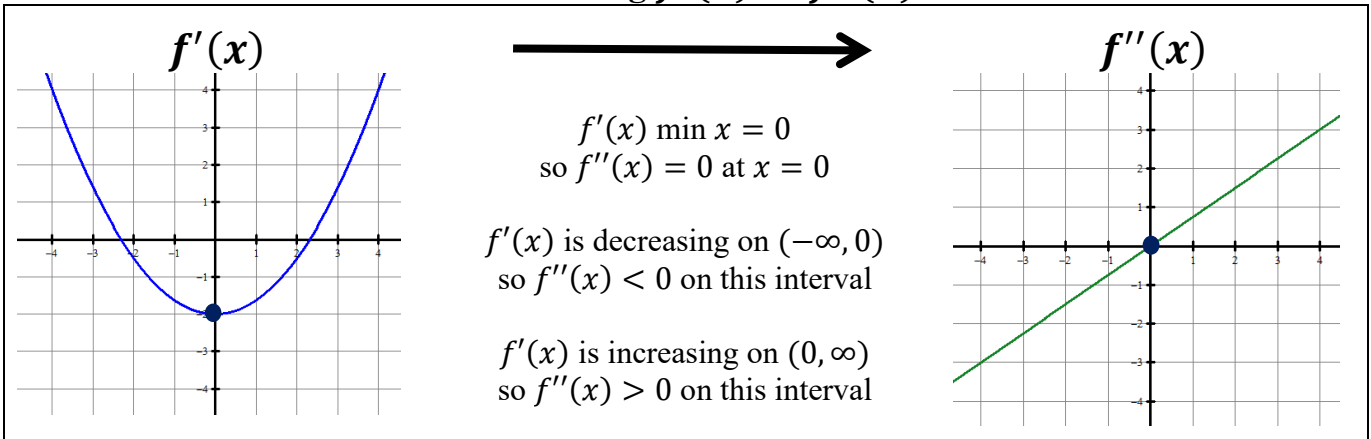
Connecting $f(x)$ to $f'(x)$



Connecting $f(x)$ to $f''(x)$

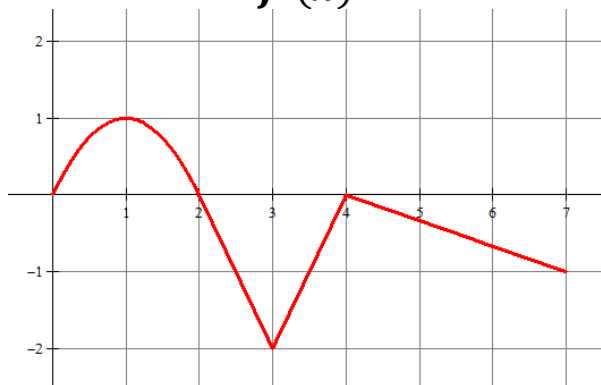


Connecting $f'(x)$ to $f''(x)$



Using $f'(x)$ to draw conclusions about $f(x)$

$f'(x)$



Find Extrema of $f(x)$

$x = 2$ and 4 are critical points because
 $f'(x) = 0$

$x = 2$ is a maximum because
 $f'(x)$ is positive on left, negative on right

$x = 4$ is NOT an extrema because
 $f'(x)$ is negative on left, negative on right

Find Points of Inflection of $f(x)$

$x = 1, 3,$ and 4 are possible points of inflection
because
 $f''(x) = 0$ or *DNE*

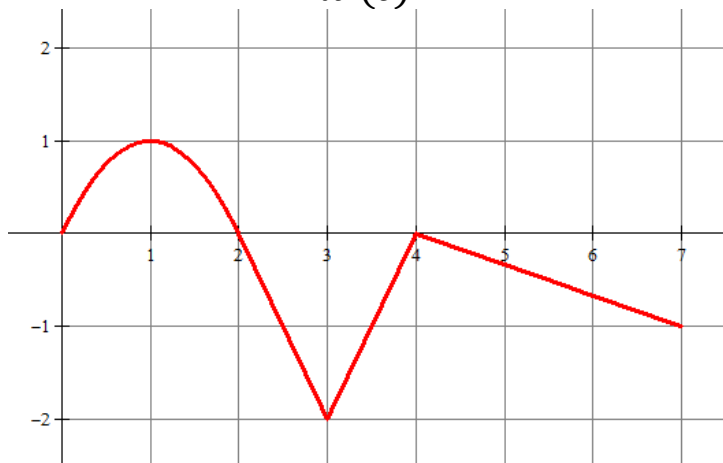
$x = 1$ is a point of inflection because
 $f''(x)$ changes sign from positive to negative at
 $x = 1$.

$x = 3$ is a point of inflection because
 $f''(x)$ changes sign from negative to positive at
 $x = 3$.

$x = 4$ is a point of inflection because
 $f''(x)$ changes sign from positive to negative at
 $x = 4$.

Now interpret the same graph as linear motion if the graph represents velocity of a particle moving along x -axis.

$x'(t)$



Moving Right or Left?

Particle moves right on $(0,2)$.

$t = 2$ particle changes direction.

Particle moves left on $(2,4)(4,7)$.

$t = 4$ particle has no velocity.

The maximum speed happens at $t = 3$.

Speeding up or Slowing down?

Particle speeds up on $(0,1)$
because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on $(1,2)$
because $f'(x)$ has a different sign from $f''(x)$

Particle speeds up on $(4,7)$
because $f'(x)$ has the same sign as $f''(x)$

Particle speeds up on $(2,3)$
because $f'(x)$ has the same sign as $f''(x)$

Particle slows down on $(3,4)$
because $f'(x)$ has a different sign from $f''(x)$